Gluon fragmentation function in polarised Λ hyperon production: The Method of Factorisation

V. Ravindran

Theory Group, Physical Research Laboratory, Navrangpura

Ahmedabad 380 009, India

Abstract

We discuss the polarised fragmentation functions of quarks and gluons in Perturbative Quantum Chromodynamics. The Altarelli-Parisi evolution equations governing these fragmentation functions are presented. We find that the first moment of the polarised gluon fragmentation function to order α_s is as important as that of the quark and anti-quark fragmentation functions. The Λ production in polarised e^+e^- annihilation where this can be realised is discussed. We propose the factorisation method to compute the gluonic contribution to Λ production to order α_s. With appropriate operator definitions for the quark and the gluon fragmentation functions, we find that the hard part of the factorisation formula is free of any infrared singularities confirming the factorisation.

Quantum Chromodynamics(QCD) is the most successful theory of strong interaction physics describing the dynamics of the quarks and gluons inside the hadrons [1]. Parton Model description of various strong interaction processes has been a successful description. Both inclusive processes such as Deep Inelastic Scattering(DIS) and hadro production in e^+e^- annihilation can well be described in the QCD improved Parton model. In DIS, the probability distribution functions of quarks and gluons inside the incoming hadrons are the non-perturbative inputs of the model. Though they are non-perturbative, their evolution in terms of the scale is completely described by the well known Altarelli-Parisi(AP) evolution equations. In the processes of hadroproduction in e^+e^- annihilation, the fragmentation functions of quarks and gluons into hadrons are the non-perturbative inputs but again their evolution is completely governed by the AP evolution equations.

Needless to say that Perturbative QCD(PQCD) improved Parton model unravelled both momentum and spin structure of the proton in terms of its constituents such as quarks and gluons. The momentum structure of the proton was studied in the unpolarised lepton proton DIS experiments [1]. Few years ago a series of polarised lepton proton DIS experiments unravelled the contributions coming from quarks and gluons to the proton spin [3]. In both the unpolarised and the polarised DIS experiments, a remarkable fact emerged which was that the gluons share significant amount of the momentum and the spin of the proton contradicting the naive expectation that most of the momentum and the spin are carried by the valence quarks. These experimental facts are well understood by using the AP evolution equations of both unpolarised and polarised quark and gluon distribution functions. These inclusive experiments are excellent tests of QCD improved Parton model and the AP evolution equations.

Recently, there has been a lot of works on the fragmentation functions of quarks and gluons measurable in the process of hadroproduction in e^+e^- annihilation and also on the test of QCD improved Parton model sublimented with the AP evolution equations for the fragmentation functions. All these analysises are on unpolarised e^+e^- scattering experiments which require only the AP equations for unpolarised fragmentation functions. The QCD improved analysis of the polarised e^+e^- scattering experiment is yet to be done.

It is in this spirit, this letter presents a systematic analysis of the polarised fragmentation functions of quarks and gluons within PQCD. This analysis is

important as we will see that polarised gluonic contribution to the production of polarised hadrons is as important as polarised quark contribution to polarised hadrons. Our analysis is quite general in the sense that it can be applied to other inclusive hadroproduction processes such as polarised proton proton and polarised proton anti-proton scattering experiments as these fragmentation functions and their evolutions are universal.

Let $D_{a(h)}^{H(s)}(x,Q^2)$ be the probability distribution for a parton a with polarisation h produced at a scale Q^2 to fragment into a hadron(H) of polarisation s carrying a momentum fraction x of the quark momentum. Now we define polarised parton fragmentation functions as

$$\Delta D_a^H(x,Q^2) = D_{a(\uparrow)}^{H(\uparrow)}(x,Q^2) - D_{a(\uparrow)}^{H(\downarrow)}(x,Q^2) \tag{1}$$

If the strong interaction world consists only of quarks and a single hadron, then the fragmentation function is $\delta(1-x)$ implying that all the spin will be transferred to the produced hadron. This is not usually the case because of two reasons. Firstly the spin of the parent parton can be distributed as it can undergo collisions which can flip the spin(mass corrections). Secondly, the single quark can fragment into many hadrons as it can excite the vacuum producing $q\bar{q}$ pairs. So the quark spin will be distributed among the produced hadrons. So it is not necessary that the quark spin is completely transferred to a single hadron. The situation is similar to DIS where the proton spin is shared by various partons.

The evolution of the quark and gluon fragmentation functions are governed by the AP evolution equations and are given by

$$\frac{d}{dt}\Delta D_{q_i}^H(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[\Delta D_{q_i}^H(y,t)\Delta P_{qq}(x/y) + \Delta D_g^H(y,t)\Delta P_{gq}(x/y)\right] \tag{2}$$

$$\frac{d}{dt}\Delta D_g^H(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_{j=1}^{2f}\Delta D_{q_i}^H(y,t)\Delta P_{qg}(x/y) + \Delta D_g^H(y,t)\Delta P_{gg}(x/y)\right] \tag{3}$$

where $t = \log(Q^2/\Lambda^2)$ and $\alpha_s(t)$ is the strong coupling constant. Here, $\alpha_s(t)\Delta P_{ab}(y)dt$ is the probability density of finding a parton of type a at the scale $t+dt$ with momentum fraction y inside the parton of type b at a scale t. The splitting functions are given by

$$\Delta P_{qq}(z) \quad = \quad C_2(R)\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right)$$

$$
\begin{aligned}
\Delta P_{gq}(z) &= C_2(R)\left(\frac{1-(1-z)^2}{z}\right) \\
\Delta P_{qg}(z) &= \frac{1}{2}(z^2-(1-z)^2) \\
\Delta P_{gg}(z) &= C_2(G)\left((1+z^4)\left(\frac{1}{z}+\frac{1}{(1-z)_+}\right)\right. \\
&\left. -\frac{(1-z)^3}{z}+\left(\frac{11}{6}-\frac{2}{3}\frac{T(R)}{C_2(G)}\right)\delta(1-z)\right)
\end{aligned}
$$

Here, $C_2(R) = (N^2-1)/2N$, $C_2(G) = N$ and $T(R) = f/2$ with $N = 3$ for $SU(3)$ and f being the number of flavours [1]. In the above equations, usual + prescription has been used. Notice that the splitting function matrix appearing in this AP evolution equation is just transpose of that appearing in the AP evolution equation for the parton probability distributions. This can be understood very easily: The emission of a quark(gluon) from a quark (gluon) only affects the probability of quark (gluon) fragmenting into hadron. On the other hand, the emission of a quark from a gluon changes the probability of the gluon fragmenting into hadron. Similarly, the emission of a gluon from a quark affects the probability of quark fragmenting into hadron. That is why this matrix is transpose. These equations can easily be solved in the Mellin space. Define

$$
\begin{aligned}
\Delta D_a^H(n,t) &= \int_0^1 x^{n-1}\Delta D_a^H(x,t)dx \\
\Delta P_{ab}(n) &= \int_0^1 x^{n-1}\Delta P_{ab}(x)dx
\end{aligned}
\tag{4}
$$

It is interesting to consider the lowest moment $n = 1$. From eqn.(3), we find that the first moment of the polarised gluon fragmentation function to order $\alpha_s(t)$ satisfies a simple first order differential equation, that is

$$\frac{d}{dt}\Delta D_g^H(t) = \alpha_s(t)\beta_0\Delta D_g^H(t) \tag{5}$$

where $\beta_0 = (11C_2(G) - 4T(R))/12\pi$. The solution to the above equation can be found very easily using renormalisation group(RG) equation for the QCD coupling constant,

$$\frac{d}{dt}\alpha_s(t) = -\beta_0\alpha_s^2(t) \tag{6}$$

From eqns.(5, 6), we obtain an interesting behaviour of first moment of gluon fragmentation function: the product of the first moment of polarised gluon fragmentation function times the strong coupling constant is scale independent to order $\alpha_s^2(t)$,

$$\frac{d}{dt}(\alpha_s(t)\Delta D_g^H(t)) = 0(\alpha_s(t)^3) \tag{7}$$

In other words, to order $\alpha_s^2(t)$, ΔD_g^H increases as the scale t increases, i.e

$$\Delta D_g^H(t) = K \log\left(\frac{Q^2}{\Lambda^2}\right) \tag{8}$$

where K is some constant. Recall that the counter part of such a relation in the polarised parton distribution functions exists and has opened up a better understanding of the spin structure of the proton [3]. That is, due to the similar relation

$$\frac{d}{dt}(\alpha_s(t)\Delta g(t)) = 0(\alpha_s(t)^2) \tag{9}$$

where $\Delta g(t)$ is the first moment of polarised gluon distribution function, the polarised gluonic contribution to proton spin is significant at very high energies. This was experimentally observed and the result was that the gluon carries significant amount of the spin of the proton. Hence the relation(eqn.(7)) suggests that at very high energies the polarised gluon fragmentation into polarised hadrons is as significant as polarised quark fragmentation into polarised hadrons. So care should be taken when one interprets the data on polarised hadron production in polarised e^+e^- annihilation processes.

Now let us consider the first moment of the polarised quark fragmentation function into polarised hadron. From eqn.(2), it turns out

$$\frac{d}{dt}\Delta D_q^H(t) = \frac{1}{\pi}\alpha_s(t)\Delta D_g^H(t) \tag{10}$$

From eqns. (7) and (10), we find that $\Delta D_{q_i}^H(t)$ grows as t. So this is a remarkable behaviour. This is very different from first moment of the polarised quarks in polarised hadron where it is scale independent.

It is instructive to find out the consequences of these results from theoretical point of view. Let us consider the polarised inclusive hadroproduction of e^+e^- scattering. In the quark sector, the first moment of the cross section is

proportional to $\Delta D_q^H(t)$. The eqn. (10) suggests that the leading behaviour of $\Delta D_q^H(t)$ is logarithmic. In the gluonic sector, the first moment of the gluonic contribution is proportional to $\Delta D_g^H(t)$. The coefficient function is proportional to $log(Q^2)$ times strong coupling constant $\alpha_s(t)$. So, from the eqn. (7), the first moment of the gluonic contribution is of the same order as that of leading order quark contribution. Hence the naive expectation that polarised gluonic contribution to polarised hadroproduction is next to leading order effect is not correct. The above observation is very general as it is based on the AP evolution equation for polarised parton fragmentation functions. So the above analysis is applicable to other polarised hadroproduction processes.

In the following we consider the polarised hadroproduction in polarised e^+e^- annihilation [4]. Since the gluonic contribution is as important as quark contribution, we compute the polarised gluonic contribution to polarised Λ production. We work in the energy range where only process where 'photon' fragmenting into hadron is dominant. The reason why we consider inclusive polarised Λ production is that the cross section is large compared to that of other particles. Also it is easy to measure the polarisation of Λ [5]. Our analysis can be extended to other hadro productions. The complete analysis including Z exchange channel is reserved for future publication [6].

The inclusive polarised Λ production rate factorises as:

$$d\sigma(s,s_1) = \frac{1}{4q_1.q_2} L^{\mu\nu}(q_1,q_2,s_1)(\frac{e^2}{Q^4}) 4\pi W_{\mu\nu}^{\Lambda}(q,p,s) \frac{d^3p}{(2\pi)^3 2p_0} \tag{11}$$

where $L_{\mu\nu}(q_1,q_2,s_1)$ is the leptonic part arising from e^+e^- annihilation into a photon of virtuality Q^2 and $W_{\mu\nu}^{\Lambda}(p,q,s)$ is photon fragmentation tensor. The arguments of these tensors are momenta described in the figures 1 and 2 and s, s_1 are the spins of Λ and electron respectively. The photon fragmentation tensor contains all the information about the polarised quark and gluon fragmentation into polarised Λ.

The operator definition of $W_{\mu\nu}^{\Lambda}(q,p,s)$ is found to be

$$W_{\mu\nu}^{\Lambda}(q,p,s) = \frac{1}{4\pi}\int d^4\xi e^{iq.\xi} < 0|J_\mu(0)|\Lambda(p,s)X >< \Lambda(p,s)X|J_\nu(\xi)|0 > \tag{12}$$

where $J_\mu(\xi)$ is the electromagnetic current, X is unobserved hadrons (where the summation over all X is assumed), q is the virtual photon momentum

given by $q = q_1 + q_2$, p and s are the momentum and spin of the polarised Λ detected.

The polarised Λ production cross section gets contribution only from antisymmetric parts of leptonic and photonic tensor. The antisymmetric part of the leptonic tensor is found to be [7]

$$\tilde{L}_{\mu\nu}(q_1, q_2, s_1) = -2ie^2 s_1 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \tag{13}$$

where again q_1, q_2 are the momenta of the incoming leptons and s_1 is the spin of the polarised lepton(electron). The photonic fragmentation tensor is not calculable in Perturbative QCD(PQCD) as we do not know how to compute the matrix element of em(electro magnetic) current between Λ states and the vacuum. But this can be parametrised using Lorentz covariance, gauge invariance, Hermiticity, and parity invariance. Hence the antisymmetric part of this photon fragmentation tensor takes the following form [7]:

$$\tilde{W}^\Lambda_{\mu\nu}(q, p, s) = \frac{i}{p.q} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma \hat{g}_1^\Lambda(x, Q^2) + \frac{i}{p.q} \epsilon_{\mu\nu\lambda\sigma} q^\lambda \left(s^\sigma - \frac{s.q}{p.q} p^\sigma \right) \hat{g}_2^\Lambda(x, Q^2) \tag{14}$$

where $x = 2p.q/Q^2$, $Q^2 = q^2$ and $s^2 = -1$. Here the polarised fragmentation functions $\hat{g}_i^\Lambda(x, Q^2)$ are real and Lorentz invariant, hence they are functions of x and Q^2.

We consider the following asymmetric cross section such that only the $\hat{g}_1^\Lambda(x, Q^2)$ structure function is projected out:

$$\frac{d\sigma(\uparrow\uparrow - \uparrow\downarrow)}{dxdCos(\theta)} = \alpha^2 \frac{\pi}{Q^2} x \hat{g}_1^\Lambda(x, Q^2) Cos(\theta) \tag{15}$$

Where $\alpha = e^2/4\pi$, θ is the angle between produced hadron and the incoming electron. Here, $\uparrow\uparrow$ means that both incoming electron and the produced hadron are paralelly polarised and $\uparrow\downarrow$ means that they are polarised antiparalelly. From the above asymmetry it is clear that the polarised fragmentation function $\hat{g}_1^\Lambda(x, Q^2)$ can be measured only through angular distribution of polarised Λ [4].

Recall that in the DIS, the hadronic tensor appearing in the cross section formula is factorised into a perturbatively calculable hard part and non-perturbative matrix elements of quark and gluon operators sandwiched between hadronic states(called soft part) [8]. This factorisation ensures that

the hard part contains no IR singularities. Following similar factorisation procedure, one can write the photonic fragmentation tensor in terms of completely calculable hard part denoted by $H^a_{\mu\nu}(y,Q^2)$ and non-perturbative operator matrix elements denoted by $D^\Lambda_a(z,Q^2)$ whose operator definitions will be given later. Hence, the factorisation formula for photon fragmentation function reads as

$$\tilde{W}^{\Lambda(s)}_{\mu\nu}(x,Q^2) = \sum_{a(h)} \int_x^1 \frac{dy}{y} \mathcal{H}^{a(h)}_{\mu\nu}(x/y,Q^2) \Delta D^{\Lambda(s)}_{a(h)}(y,Q^2) \tag{16}$$

where a runs over all the partons such as quarks, anti-quarks and gluons, h is the helicity of the parton. $H^{a(h)}_{\mu\nu}(z,Q^2)$ is the hard part of the parton differential cross section for the production of parton of type a with polarisation h and energy fraction $z = 2p.q/Q^2$. The soft part $\Delta D^{\Lambda(s)}_{a(h)}(y,Q^2)$ is the usual polarised parton fragmentation function whose operator definition and the physical interpretation in terms of the partons will be discussed in the following. The above factorised formula ensures that the $H^a_{\mu\nu}(z,Q^2)$ is free of any Infrared(IR) singularities. In other words, if $H^a_{\mu\nu}(z,Q^2)$ is free of any IR singularities, the tensor $W^\Lambda_{\mu\nu}(x,Q^2)$ is said to be factorisable. In the following we shall show that such a factorisation exists and that it works to order α_s in the gluonic sector. This is the by product of our analysis on the evaluation of first moment of polarised gluonic contribution to polarised Λ production.

Let us first work out the operator definition for $D^\Lambda_q(x,Q^2)$ to leading order. As we know the photon fragmentation tensor $W^\Lambda_{\mu\nu}(x,Q^2)$ is not computable in the perturbation theory due to the non-perturbative nature of hadrons(here Λ). The physical interpretation of the photon fragmentation tensor in terms of the parton fragmentation functions can be easily got if we work on the light cone [9] since we are interested in the limit $Q^2 \to \infty$(see below). To make the computation simple and the physics more transparent, let us assume that the currents appearing in the photon fragmentation tensor are free field currents so that formal manipulations can be done with out explicitly evaluating them between the Λ states and the vacuum. Note that dominant contribution to this integral in $W^\Lambda_{\mu\nu}(x,Q^2)$ (eqn.(12)) in the limit $Q^2 \to \infty$ comes from the light cone region $\xi^2 \to 0$, otherwise the exponential factor will kill the integral in this limit. In this lightcone limit $\xi^2 \to 0$, let us assume that only one of the field operators (say quark) of each current is responsible for the fragmentation whatso ever be the mechanism. Hence the

rest of the operator matrix elements can be computed between free parton (say anti-quark) states to the leading order. After a little algebra we find that the quark contribution to the structure function $g_1^\Lambda(x,Q^2)$ to leading order on the light cone turns out to be

$$g_1^\Lambda(x,Q^2) = 3\frac{1}{x}\sum_q e_q^2 \Delta D_q^{0\Lambda}(x,Q^2) \tag{17}$$

where

$$\Delta D_q^{0\Lambda}(x,Q^2) = \frac{x}{8\pi}\int d\xi^- e^{-i\xi^- p^+/x} < 0|\gamma_+\gamma_5\psi(0)|\Lambda X >< \Lambda X|\bar{\psi}(\xi^-)|0 > \tag{18}$$

where average over colour is implicit in the eqn.(18). Similarly one can find out the anti-quark contribution to $g_1^\Lambda(x,Q^2)$. Following the procedure adopted in reference [10], we find that the $\Delta D_q^{0\Lambda}(x,Q^2)$ measures the number of up($\uparrow$) polarised quarks fragmenting into up($\uparrow$) polarised Λ minus the number of down($\downarrow$) polarised quarks fragmenting into up($\uparrow$) polarised Λ.

Generalising the above idea, one can express polarised quark, anti-quark and gluon fragmentation functions in terms of quark and gluon operators as follows [10], [11]:

$$\begin{aligned}\Delta D_{q+\bar{q}}^\Lambda(x,Q^2) &= \frac{x}{8\pi}\int e^{-i\xi^- p^+/x}\Big(< 0|\gamma_+\gamma_5\psi(0)|\Lambda X >< \Lambda X|\bar{\psi}(\xi^-)|0 > \\ &\quad + < 0|\gamma_+\gamma_5\psi(\xi^-)|\Lambda X >< \Lambda X|\bar{\psi}(0)|0 >\Big)\end{aligned} \tag{19}$$

$$\Delta D_g^\Lambda(x,Q^2) = -\frac{x^2}{4\pi p^+}\int e^{-i\xi^- p^+/x} < 0|\tilde{G}_a^{+\alpha}(0)|\Lambda X >< \Lambda X|G_{a\ \alpha}^{+}(\xi^-)|0 > \tag{20}$$

where, sum over X is assumed and $G_{\mu\nu}^a$ the usual field strength tensor. Also, averge over colour is implicit. We have also dropped the Wilson link operator as it becomes identity in the lightcone gauge $A^+ = 0$ which we choose to work with. The Λ carries the momentum p and spin s. It is clear from the above operator definitions of the parton fragmentation functions that the matrix elements of these operators for hadronic states are not calculable. But, interestingly, they are calculable for the partonic states such as quarks, anti-quarks and gluons in the perturbation theory as one knows how these partonic operators act on the partonic states. This fact that they are calculable for

the partonic states is exploited to compute the hard part ($H^a_{\mu\nu}(z,Q^2)$) of the factorisation formula.

First we compute the leading order contribution coming from the operators to the polarised fragmentation function $\hat{g}^\Lambda_1(x,Q^2)$. Using the factorisation formula and the normalisation that $\Delta D^q_q(x,Q^2) = \delta(1-x)$ to lowest order, we find that the quark and the anti-quark contributions to $\hat{g}_1(x,Q^2)$ come form Fig. 1. which we denote by $\hat{g}^0_1(x,Q^2)$,

$$\hat{g}^{0\Lambda}_{1q}(x,Q^2) = 3\frac{1}{x}\sum_q e_q^2\left(\Delta D^\Lambda_q(x,Q^2) + \Delta D^\Lambda_{\bar{q}}(x,Q^2)\right) \tag{21}$$

From the above equation, it is clear that the polarised fragmentation function $g^\Lambda_1(x,Q^2)$ to leading order $\alpha_s(Q^2)^0$ measures the polarised quark and anti-quark fragmentation functions weighted by appropriate charge square factors(e_q^2).

The $\alpha_s(Q^2)$ corrections to $g^\Lambda_1(x,Q^2)$ come from two sources: **1**. gluon bremstalung and virtual corrections to polarised quark and anti-quark fragmenting into polarised Λ, **2**. the polarised gluon fragmenting into polarised Λ. Here, we are interested only in the evaluation of polarised gluonic contribution to the production. Using the factorisation formula(eqn.(16), the gluonic contribution to $g^\Lambda_1(x,Q^2)$ can be formally written as

$$\hat{g}^\Lambda_{1g}(x,Q^2) = \frac{1}{x}\sum_q e_q^2\int_x^1\frac{dy}{y}\mathcal{H}_g(x/y,Q^2)\Delta D^\Lambda_g(y,Q^2) \tag{22}$$

We again use the factorisation formula to compute $H^g_{\mu\nu}(z,Q^2)$. $H^g_{\mu\nu}(z,Q^2)$ gets contribution from order $\alpha_s(Q^2)$ onwards. To compute this, we replace polarised Λ by polarised gluon in the eqn. (16). With this replacement we find that the left hand side of the eqn. (16) is just the cross section for photon decaying into polarised gluon and a quark-antiquark pair. On the other hand, the right side involves the evaluation of quark operator for polarised gluonic states to order $\alpha_s(Q^2)$ and $g^q_{1q}(z,Q^2)$ and $\Delta D^g_g(z,Q^2)$ to lowest order $(\alpha_s(Q^2)^0)$. Hence,

$$\mathcal{H}_g(z,Q^2) = W_g(z,Q^2) - 3\Delta D^g_q(z,Q^2) \tag{23}$$

Let us now compute $W_g(z,Q^2)$ appearing in the eqn.(23). $W_g(z,Q^2)$ can be computed from the polarised cross section for e^+e^- annihilating into the

polarised gluon:

$$\frac{d\sigma^g(s,s_1)}{dzdCos(\theta)} = \frac{is}{4q_1.q_2}\tilde{L}_{\mu\nu}(q_1,q_2,s_1)\frac{1}{Q^4}\epsilon_{\mu\nu\lambda\sigma}q^\lambda p_g^\sigma \frac{Q}{p_g.q}H_g(p_g,q) \tag{24}$$

where, $z = 2p_g.q/Q^2$, p_g is the momentum of the polarised gluon and θ is the angle between produced gluon and the incoming electron. $H_g(p_g,q)$ appearing in the above equation (eqn. (24)) is found to be

$$H_g(p_g,q) = \frac{Q}{32(2\pi)^3}\int dx_2 \mathcal{P}_g^{\mu\nu}|M^g|^2_{\mu\nu} \tag{25}$$

where $x_2 = 2p_q.q/Q^2$ and the projector $\mathcal{P}_g^{\mu\nu} = i\epsilon_{\mu\nu\lambda\sigma}p_g^\lambda q^\sigma/2p_g.q$. The projected matrix element square $\mathcal{P}_g.|M^g|^2$ is computed from the Fig. 2(with gluon polarised) and is given by

$$\begin{aligned}\mathcal{P}_g^{\mu\nu}|M^g|^2_{\mu\nu} \;=\; & 3C_2(R)\frac{4(2\pi)^3e_q^2\alpha\alpha_s}{\pi}\left(\frac{m_g^2Q^2-st}{(s+t)^2}\right)\left[2\frac{(s+t)(s+t-m_g^2-Q^2)}{st}\right.\\ &+\frac{4m_g^2Q^2-2m_g^2s-2Q^2s+s^2-t^2}{t^2}\\ &\left.+\frac{4m_g^2Q^2-2m_g^2t-2Q^2t+t^2-s^2}{s^2}\right]\end{aligned} \tag{26}$$

where the Mandelstam variables $s = (p_{\bar{q}}+p_g)^2$,$t = (p_q+p_g)^2$ and $u = (p_q+p_{\bar{q}})^2$ satisfy $s+t+u = m_g^2+Q^2$. Also note that $s = Q^2(1-x_2)$. Substituting the eqns. (26) and (25) in the eqn. (24), we find that the asymmetry in the polarised gluon emission turns out to be

$$\frac{d\sigma^g(\uparrow\uparrow-\uparrow\downarrow)}{dzdCos(\theta)} = \frac{3C_2(R)e_q^2\alpha^2\alpha_s}{Q^2}\left[(2-z)\log\left(\frac{1+\beta_z}{1-\beta_z}\right)-2(2-z)\beta_z\right]Cos(\theta) \tag{27}$$

where $z = 2p_g.q/Q^2$, $\beta_z = (1-4m_g^2/Q^2z^2)^{1/2}$ and θ here is the angle between the outgoing polarised gluon and incoming lepton(electron). The above result shows that there are no soft singularities. The small nonzero gluon mass is used to regulate collinear divergence. Since we are looking at the polarised gluon production, there is no virtual correction to this order, hence there is no UV divergences. From the eqn. (27), we find that

$$W_g(z,Q^2) = 3C_2(R)\frac{\alpha_s}{2\pi}2\left[(2-z)\log(\frac{Q^2z^2}{m_g^2})-2(2-z)\right] \tag{28}$$

Now, let us compute the matrix element $D_q^g(z,Q^2)$ appearing in the eqn. (23). We can compute this using the eqn. (19) with Λ replaced by polarised gluon. The diagrams contributing to this matrix element are given in Fig.3. This is found to be

$$\begin{aligned}\Delta D_{q+\bar{q}}^g(z,Q^2) &= 4C_2(R)\alpha_s z \int \frac{d^dk}{(2\pi)^d} 2\pi\delta(k^2)2\pi\delta(k^+ - p_g^+ + p_g^+/z) \quad (29)\\ &\quad Tr\left[\gamma^+\gamma_5 \frac{\not{k} - \not{p}_g}{(k-p_g)^2 - i\epsilon} \not{\epsilon}^* \not{k} \not{\epsilon} \frac{\not{k} - \not{p}_g}{(k-p_g)^2 + i\epsilon}\right]\end{aligned}$$

The above expression is UV singular and we regulate this divergence in Dimensional Regularisation method. A small gluon mass m_g is introduced to regulate IR divergence. From the detla functions appearing in the above equation we find that $k^+ = p_g^+(z-1)/z$ and $k^- = k_\perp^2 z/(2p_g^+(z-1))$. Using these constraints, the k^+ and k^- integrations can be safely done. The $k_\perp^2$ integration can also be done and the result is

$$\Delta D_{q+\bar{q}}^g(z,Q^2) = C_2(R)\frac{\alpha_s}{2\pi}2\left[(2-z)\log\left(\frac{\mu_R^2 z^2}{mg^2(1-z)}\right) - z\right] \quad (30)$$

The UV divergence appearing in the eqn.(30) is renormalised in $\bar{M}S$ scheme, hence the appearance of substration scale μ_R^2.

Substituting the eqn.(30) in eqn. (23), we find that the gluonic coefficient function is

$$\mathcal{H}^g(z,Q^2) = 3C_2(R)\frac{\alpha_s}{2\pi}2\left[(2-z)\log\left(\frac{Q^2}{\mu_R^2}\right) + (2-z)\log(1-z) + 3z - 4\right] \quad (31)$$

where μ_R is the scale at which the gluon fragmentation operator matrix element is renormalised. From the above equation it is clear that the collinear divergence appearing in the cross section formula cancels against that appearing in the matrix element leaving the hard part $H_g(z,Q^2)$ free of any IR divergences. This is the proof of factorisation in the gluonic sector to order $\alpha_s(Q^2)$. Substituting the above equation in eqn.(22), we get the gluonic contribution to $g_1^\Lambda(x,Q^2)$ and hence the gluonic contribution to the asymmetry given in the eqn. (15).

In this paper we have systematically analysed the importance of gluons to the production of polarised Λ in an inclusive e^+e^- annihilation. We have done

this using the AP equation for the gluon fragmentation function. We have computed the gluonic contribution to order α_s to the polarised fragmentation function $g_1^{\Lambda}(x, Q^2)$ appearing in the asymmetry of the polarised cross section using the factorisation formula. The factorisation that the cross section can be factored into a hard and a soft part has been demonstrated in the gluonic sector.

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References

[1] T. Muta, Foundations of Quantum Chromodynamics, (World Scientific, Singapore, 1987).
R.D. Field, Applications of Perturbative QCD, Frontiers in Physics, (1989) Addison-Wesley Publishing Company, Redwood City, CA, USA.
R. G. Roberts, The structure of the proton (Cambridge University Press, Cambridge, 1990).
G. Altarelli, Phys. Rep. **81** (1980) 1.

[2] B. Mele and P. Nason, Nucl. Phys. **B361** (1991) 626,
P. Nason and B.R. Webber, Nucl. Phys. **B421** (1994) 473 and the references therein.

[3] M. Anselmino, A. Efremov, and E. Leader, Phys. Rep. **261** (1995) 1.

[4] R. Ramachandran, R.P. Bajpai, Phys.Lett. **115B** (1982) 313.
H.S. Mani, R. Ramachandran, Phys. Rev.**D28** (1983), 1774.
M. Burkardt and R.L. Jaffe, Phys. Rev. Lett. **70** (1993) 2537.
R.L. Jaffe, **MIT-CTP-2534** HUTP-95/A018.

[5] B. Lundberg *etal.*, Phys. Rev. **D 40**, (1989) 3557, and the papers cited thereina,
P. Hansen and U. Dickow, Report No. **ALEPH 95-055** .

[6] V. Ravindran, under preparation.

[7] W. Lu, Phys. Rev. **D51** (1995) 5305.
W. Lu, X. Li and H. Hu, Phys. Rev. **D55** (1995) 131.

[8] Perturbative QCD, World Scientific Edited by H. Mueller.

[9] R.L. Jaffe, **MIT-CTP-2506** HUTP-96/A003.

[10] R.L. Jaffe, in Relativistic Dynamics and Quark-Nuclear Physics,
M.B. Johnson and A. Picklesimer, eds. (Wiley, New York, 1986).

[11] X. Ji, Phys. Rev. **D49** (1994) 114.

Figure Captions:

1. Graph contributing to $e^-(q_1)e^+(q_2) \to \bar{q}(p_{\bar{q}})q(p_q)$.
2. Graphs contributing to $e^-(q_1)e^+(q_2) \to \bar{q}(p_{\bar{q}})q(p_q)g(p_g)$.
3. Graph contributing to $\Delta D_q^g(x, Q^2)$